Geometric Representations of Condition Queries on Three-Dimensional Vector Fields

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Abstract

Condition queries on distributed data ask where particular conditions are satisfied. It is possible to represent condition queries as geometric objects by plotting field data in various spaces derived from the data, and by selecting loci within these derived spaces which signify the desired conditions. Rather simple geometric partitions of derived spaces can represent complex condition queries because much complexity can be encapsulated in the derived space mapping itself. A geometric view of condition queries provides a useful conceptual unification, allowing one to intuitively understand many existing vector field feature detection algorithms — and to design new ones — as variations on a common theme. A geometric representation of condition queries also provides a simple and coherent basis for computer implementation, reducing a wide variety of existing and potential vector field feature detection techniques to a few simple geometric operations.

Keywords: computational fluid dynamics, feature detection, flow visualization, selective visualization.

1 Introduction

Three-dimensional vector fields are hard to visualize in their entirety because they contain relatively high-dimensional entities in a relatively high-dimensional space. Discretized fields of N nodes with three-component vectors live in a $3N$-dimensional configuration space, $\mathbb{R}^{3N}$. For even small values of N, the bandwidth required to display certain features of such high-dimensional entities can easily overwhelm that of a two-dimensional display.

It is natural to resort to *selective visualization* [7] of high-dimensional geometric objects, to reveal various features or aspects of their structure. For vector fields, one can make selections based on location in the field, or on particular geometric or topological conditions manifested by the vectors and their distribution in the field. Selective visualization of vector fields is tantamount to querying locations or conditions throughout the field, and then somehow displaying the results of that query. A *location query* on distributed data asks for the conditions at a particular location; a *condition query* asks where particular conditions are satisfied [2].

Most scientific visualization techniques favor location queries over condition queries — that is, the techniques generally allow one to select field locations more readily than field conditions, and the techniques generally display the conditions at a given location more effectively than they indicate the locations at which given conditions hold. Techniques involving isosurfaces are the obvious exception to this rule.

This paper attempts to provide a general framework for thinking about the types of condition queries applicable to three-dimensional vector fields for the purposes of scientific visualization. Actually the treatment will not be all that general, and will instead be heavily biased towards a few rather parochial examples, taken from the fields of computational fluid dynamics. This restriction in scope is largely unavoidable, since the sorts of condition queries which prove useful depend on the sorts of data on hand. Commonly, scientific visualization techniques that rely upon condition queries are known as *feature detection* or *feature extraction* techniques; and the notion of what constitutes an interesting feature is domain specific.

The approach taken here is geometrically motivated. The primary unifying theme is to look at vector field data, and quantities derived from the data, in spaces derived from the data. By itself, the distribution of the data in these "derived spaces" can be informative. In addition, geometric partitioning in appropriate derived spaces can select data subsets based on particular data attributes, and therefore allows one to represent condition queries as geometric objects.

Representing condition queries as geometric objects is useful for two reasons. First, it provides a useful conceptual unification, allowing one to intuitively understand a wide range of existing feature detection techniques as variations on a common theme, and allowing one to imagine new techniques as further thematic modifications. Second, the geometric viewpoint provides a simple and coherent basis for computer implementation — a few basic geometric operations can support a varied array of analytic and exploratory vector field techniques.

Both conceptually and practically, representing condition queries geometrically can be more intuitive and less cumbersome than purely algebraic representations, or more "semantic" methods, using a custom query language. The geometric approach provides a powerful and general way to analyze and explore three-dimensional vector fields with condition queries.

2 Condition queries on vector fields

Condition queries on field data ask where particular conditions hold. A visualization technique which allows a user to effectively pose and answer a condition query must deal with three distinct sorts of data.

1. First, there is the field which is being queried. We shall refer to this as the *target field*. In the present work, target fields will be three-dimensional vector fields.

2. Second, there is the query itself. The query consists of a set of conditions that are evaluated at each target field location. The conditions may be constant, or they may vary with target field location.

3. Third, there is the *result* of applying the query to the target field. The result consists of the set of target field locations which satisy the conditions posed in the query. If the target field is a three-dimensional vector field, the result can be any collection of 0D, 1D, 2D, or 3D loci.

Depending on the identity of the target field, and the nature of the query, one may attach a particular significance or interpretation to the result, and perhaps even recognize it as a "feature". For this

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reason, certain types of condition queries are referred to as *feature detection* or *feature extraction* techniques. For example, a condition query may identify a "vortex core" or "separation surface".

In this paper we shall not say a whole lot about how the final results of condition queries are displayed, or about other things that might be done with them. Van Walsum and Post [7] have outlined some possibilities for using the results of condition queries in ways other than direct visualization. We shall be primarily concerned here with the condition queries themselves, and how we can represent [conceive of] them as geometric objects.

2.1 Uniform vs nonuniform condition queries

There is a primary distinction between *uniform* condition queries, which are constant with respect to target field location, and *nonuniform* condition queries, which vary with target field location. A uniform condition query applies precisely the same local test at each target field location. A nonuniform condition query applies the same *type* of test at each target field location, but the actual local test which is carried out is location dependent.

An important possibility is that nonuniform condition queries can be locally derived — that is, data from a target field location can itself be used to generate the conditions against which the local target field value is tested. Local derivations may also produce logical branch points, from which different types of condition queries are posed depending on the local context.

2.2 Geometry of data and geometry of queries in derived spaces

Discretized field data can be plotted in various spaces, where the coordinates are derived from the dependent (usually nonspatial) variables of the dataset. Field points with similar derived values will map to similar locations in these "derived spaces" [2], regardless of their original relative spatial locations. Mapping the entire field into a derived space results in an *image* of the dataset. Certain features of derived space images — including overall ranges, patterns of distribution, and coherent structures — graphically depict the derived quantity values of the underlying data, and can be very informative.

The fundamental idea developed here is that partitions of derived spaces, which select some range of derived quantities, can be used to represent condition queries geometrically. More specifically, a derived space can be partitioned into "pass" and "fail" regions (not necessarily contiguous or simply connected), and a target field location can then be conditionally judged according to the derived space region in which it falls. We shall refer to those regions of the derived space which signify a positive outcome of the condition query as the *condition loci*. For a uniform condition query, a single partition of the derived space is used for all target field locations. For a nonuniform condition query, a new partition (and perhaps even a new derived space) is constructed at each target field location, usually incorporating local information.

Many derived spaces have geometrically significant regions or a natural structure which allows them to be partitioned *a priori* for certain kinds of condition queries. There is also the possibility of target field *data driven* partitioning of a derived space. This is the usual case for nonuniform condition queries, in which local target field data are used to synthesize a local test. Even for uniform condition queries, specification of the single globally applied derived space partition might be guided by features of the derived space image. In either case, rather simple geometric partitions of derived spaces can represent complex condition queries, *because much complexity can be encapsulated in the derived space mapping itself.*

There are a multitude of derived spaces, of various dimensions, in which one could usefully map field data. For the three-dimensional vector field data considered here, we will focus on geometric representations of condition queries in three three-dimensional derived spaces: (1) vector component space, (2) the space of the three primary vector gradient tensor invariants, commonly referred to as PQR space, and (3) local linearized vector spaces.

2.3 Vector component space queries

A vector component space (or just "component space") is a space spanned by the basis vectors of some coordinate system. This is just a vector space, but we add (or substitute) the term "component" to emphasize that when we are considering vectors from fields, we ignore their spatial coordinates, and use only their components to locate them in this (minimally) derived space. In other words, all vectors from a field are rooted at the origin in vector component space. For three-dimensional vector fields, the vector component space is just $\mathbb{R}^3$. In the following we will assume the three-dimensional vector component space to be spanned by the Cartesian basis vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$, though a cylindrical $(\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_z)$ or spherical $(\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$ basis is sometimes more natural.

2.3.1 Simple geometric queries in component space

The basic attributes of vectors are magnitude and direction. A geometric test on a vector asks whether its magnitude and/or direction falls within certain ranges. A geometric condition query on a vector field applies either a constant or variable magnitude/direction test at all points in the target field. Both uniform or nonuniform geometric condition queries on three-dimensional vector fields can be posed as partitions of a three-dimensional vector component space, simply by specifying which regions of component space satisfy the condition — that is, by geometrically describing the condition loci.

Simple geometric queries are represented by simple geometries in component space. For example, the condition locus for querying vectors having magnitudes within some range is a spherical shell (or "hollow sphere") centered about the origin. The condition locus for selecting vectors within some range of directions symmetrically disposed about a central axis is a cone with its apex at the origin. A condition locus specifying both magnitude and direction ranges is the intersection of a spherical shell and a cone — very nearly a frustum of a cone, whose parallel faces are replaced by concentric spherical sections.

Purely directional condition queries can be represented on a two-dimensional subspace of the component space, a unit sphere sometimes called the direction sphere, or the Gauss sphere. A range of directions is represented as a disk on the sphere. In the limit as the disk is shrunk to a point, a uniform condition query yields *isodirection lines*, along which the vector field is parallel to a fixed reference vector. In three-dimensional vector fields, isodirection lines are one-dimensional loci, which can end only on critical points or domain boundaries.

2.3.2 Representing directional queries as vector fields

Important cases arise when the condition locus is a vector subspace of the component space. In $\mathbb{R}^3$, the vector subspaces are a point at the origin (0D), a line through the origin (1D), and a plane through the origin (2D).

In the case where the condition locus is a single point at the origin in component space, one is querying for target vector field locations with zero magnitude and unspecified direction, i.e., critical points.

In the case where the condition locus is a line through the origin in component space, one is querying for vectors which are either parallel or antiparallel to a given direction. The line can be thought

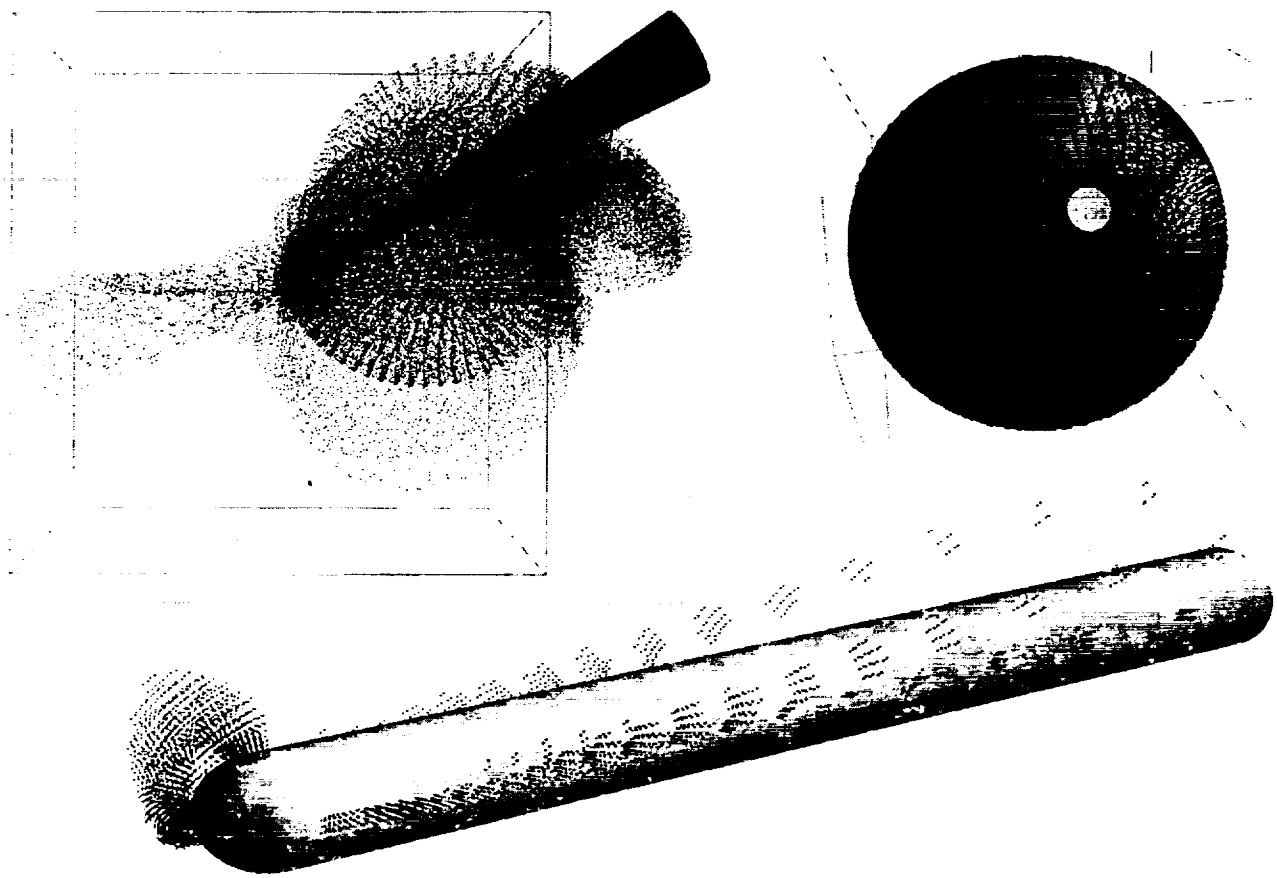

Figure 1: (Top left) Hemispherical cylinder flow dataset plotted in velocity component space, with a directional condition query represented as a cone. The apex of the cone is at the origin, and the freestream maps to the right side of the axis-aligned box 19° above the positive x-axis, which extends to the right. The "nose" protruding from the box on the right indicates accelerated flow, and the "ponytail" extending to the left of the origin indicates flow reversal. (Top right) Same as left image, but the view is more head-on to the x-axis, and all velocity vectors have been normalized so they lie on the unit "direction sphere". The same directional condition query is now a disk, or "spotlight" on the sphere, somewhat above the image of the freestream. (Bottom) Results of the condition query in top images, showing that flow is deflected upward at the "forehead" of the hemispheric cylinder, and is also being swept upward into the primary vortices that extend along either side of the top of the cylinder (see Figure 4).

of as a pair of antipodal points (or "poles") on the direction sphere, or as the limiting case of an opposed pair of narrowing direction cones in the full three-dimensional component space. One can test if a vector lies in a given 1D vector component subspace by asking if it is **parallel** or **antiparallel** to a single representative from that subspace, which we can call a reference tangent vector. One may wish to restrict the directional test to simple parallelism, in which case the condition locus is a half-line or ray anchored at the origin of component space (and no longer a vector subspace), or a single point on the direction sphere.

In the case where the condition locus is a plane through the origin in component space, one is querying for vectors with one directional degree of freedom restricted. The plane can be thought of as a great circle (an "equator") on the direction sphere. A plane through the origin in component space can be represented by a reference normal vector, and we can ask if a vector lies in that plane by asking if it is **perpendicular** to the reference normal vector.

Thus one can represent a test of membership of a given target field vector in either a 1D or 2D vector component subspace with a single reference vector and an accompanying condition of parallelism or perpendicularity. The set of all reference vectors, which figure in relative directional tests over the entire target field, is itself a vector field, which we shall refer to as the *test vector field*. Since neither vector parallelism nor perpendicularity is affected by magnitude, a given relative directional condition query is represented by an entire equivalence class of vector fields, all of which share the same local tangent vectors. Also note that since parallelism and perpendicularity are commutative relations, the roles of target and test fields in these kinds of tests are interchangeable.

In general, the loci where two three-dimensional vector fields are parallel are one-dimensional space curves. The loci where two three-dimensional vector fields are mutually perpendicular are generically two-manifolds, or surfaces. Both the parallelism and perpendicularity tests do of course admit of higher-dimensional results, and although these are exceptional or degenerate cases they may be important, e.g. the perpendicular velocity and vorticity fields of complex lamellar flows in two-dimensions or shear layers, and the parallel velocity and vorticity fields of Beltrami flows in magnetohydrodynamics.

A *constant* test vector field and the condition of parallelism make up a uniform directional query yielding isodirection lines. A uniform condition query for perpendicularity will produce a sort of dual to the isodirection lines — surfaces ("isonormal surfaces") in the target vector field where a given direction (the reference normal) is absent. Nonuniform directional condition queries are represented by general test vector fields. Important cases arise when the test vector field is derived from the eigenvectors or eigenvector plane normals of the target vector gradient tensor. Directional tests involving these quantities can be particularly informative because they reveal how the target vectors stand in relation to their own spatial derivatives. Such cases underlie many popular vector field feature detection algorithms — some detailed examples will

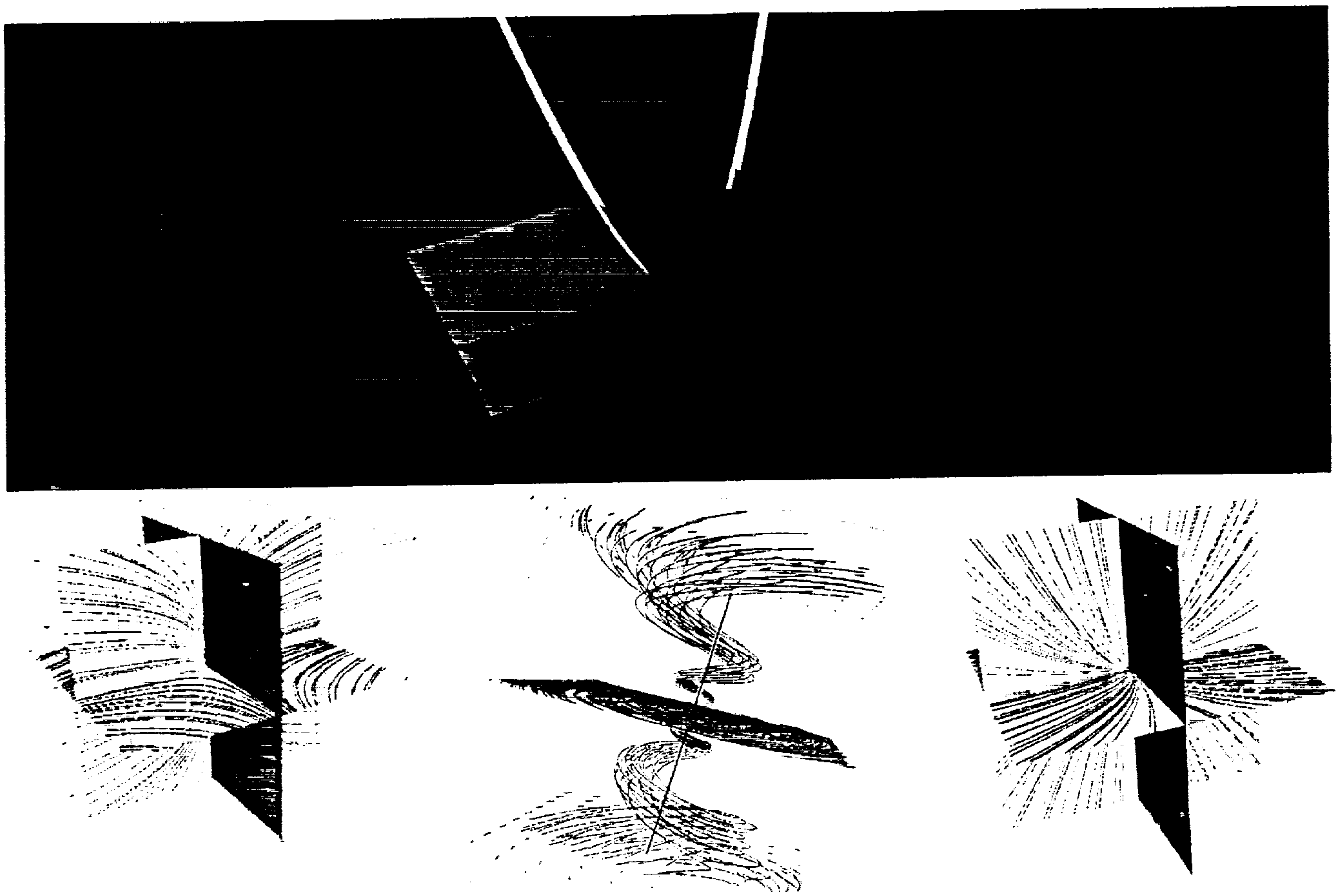

Figure 2: (Top) PQR-space, with partitions that separate different flow topologies. The white parabola lies in the PQ plane. The osculating cubic surfaces divide real and complex eigenvalues; algebraic expressions for these and the other surfaces can be found in [1]. (Bottom) Representative flow topologies, showing eigenvector planes and eigenvectors (eigenvectors are indicated by intersections of the planes in left and right cases). (left) node-saddle-saddle; (middle) swirling flow; (right) node-node-node.

be presented below.

2.3.3 General component space partitions

Vector subspaces represent a very limited set of partitions of component space. One can partition component space with affine subspaces, arbitrary space curves, surfaces, volumes, or any union or intersection of these. The utility of such condition loci in component space will in general be domain and problem specific.

For uniform geometric condition queries in particular, the distribution of the target vector field image in component space can usefully guide a conditional selection — making it possible to define *relative* condition queries, like "faster than freestream" in a flowfield. One can exploit the geometrically significant structure of component space to facilitate such "data driven" condition queries with simple geometric tools, which can be quite useful in an interactive setting for exploratory data visualization and analysis.

2.4 PQR space queries

Partitions of vector component space do not by any means exhaust the possibilities for geometric representations of condition queries on vector fields. An important class of condition queries that are not representable by partitions of vector component space involve the spatial *derivatives* of the target field vectors. These sorts of condition queries allow one to probe the topological features of a vector field.

2.4.1 Simple topological queries

The first spatial derivatives of a three-dimensional vector field make up a tensor field, which can be locally represented by a 3×3 matrix, the Jacobian matrix. The trace and determinant of this matrix, and also the sum of the determinants of its three 2×2 submatrices, yield three scalar quantities called the *principal invariants* of the tensor. These scalars are called invariants because they are unaffected by changes of coordinate system.

The principal invariants of the 3×3 Jacobian matrix are commonly referred to as P, Q, and R. These quantities can be used to construct a three-dimensional derived space, which we will call PQR-space. Each point in a three-dimensional vector field can be mapped into PQR space, where its location determines — to first order — the local topological character of the vector field. The entire PQR-space is partitioned into regions which correspond to distinct topological patterns in the underlying vector field, and these "predefined" partitions can be variously combined to represent arbitrary topological condition queries. See [1] for algebraic descriptions of the partitions, and Figure 2 for a graphical portrayal.

The image of a vector field in PQR-space can reveal topological features or patterns, which can be independently informative and can also provide an opportunity to respond to the data with data-driven condition queries. Locating a topological region or feature in a vector field may be an end in itself, but topological classifications can additionally be used as logical branch points ("conditional conditions") for further condition queries. Certain topological con-

Figure 3: Results of three algorithms for locating vortex cores, in flow around a tapered cylinder. All the algorithms rely upon the same type of nonuniform directional condition query, finding loci where the velocity field is parallel to a locally generated test vector field. (Left) Velocity parallel to vorticity. (Middle) Velocity parallel to single real eigenvector of velocity gradient tensor. (Right) Velocity parallel to single real eigenvector of vorticity gradient tensor. This latter algorithm appears more sensitive than the first two, connecting the disjoint vortex core segments into an almost unbroken helical structure, while introducing only a moderate amount of noise.

ditions are often a first cut for many feature detection algorithms.

2.4.2 Linearized field element queries

One can use the Jacobian matrix to define a linear approximation of a vector field about a given point or across a linear element —

$$\mathbf{v} = \mathbf{v_0} + \mathbf{J}(\mathbf{x} - \mathbf{x_0})$$

— where $\mathbf{x_0}$ and $\mathbf{v_0}$ are the position and vector value, respectively, at the origin of the linearized field, and $\mathbf{J}$ is the Jacobian matrix. One can always choose a coordinate system whose origin coincides with the critical point of the strictly first order field, so that the zeroth order terms ($\mathbf{v_0}$ and $\mathbf{x_0}$) vanish. This allows one to locate the image of a point or a linear element directly in the linear field $\mathbf{v} = \mathbf{Jx}$, without the arbitrary offsets imposed by an "external" coordinate system.

Locating the image of a point or element in a linearized field in terms of its "natural coordinates" is useful because the linearized field possesses a natural geometric partitioning, which allows one to pose a variety of condition queries on the point or element of the originating vector field. As discussed above, the overall topology of the linear field partitioning is given by its PQR coordinates. As discussed below, the precise geometric descriptions of the partitions are determined by the eigenvalues and eigenvectors of its Jacobian matrix.

The eigenvalues of a 3 × 3 Jacobian matrix can be (*i*) all real and all distinct, (*ii*) all real and not all distinct, or (*iii*) one real value and a pair of complex conjugates. Case (*ii*) is degenerate and will not be considered further here. If the eigenvalues are all real and distinct, then the linear field is partitioned by three axes of zero curvature trajectories (given by the eigenvectors) and three planes of zero torsion trajectories (the planes spanned by the three possible pairs of eigenvectors). The axes, and so the planes, are in general not mutually orthogonal. If the eigenvalues consist of one real value and a pair of complex conjugates, then the linear field is partitioned by a single zero curvature trajectory (in the direction of the single real eigenvector) and a single plane of zero torsion trajectories (representing the intersection of the hyperplane spanned by the complex eigenvectors with $\mathbb{R}^3$). The single real eigenvector is in general not orthogonal to the complex eigenvector plane.

The eigenvectors and eigenvector planes in the linear vector field represent important condition queries, which can be usefully exploited on any vector field for which the linear approximation is reasonably accurate. Since a trajectory on an eigenvector is uncurved, it must always stay on that eigenvector; and since all trajectories in an eigenvector planes are torsion free, they must forever stay in that eigenvector plane. For these reasons the eigenvectors and eigenvector planes are called *invariant manifolds*. The intersection of the invariant manifolds represents a critical point. Depending on the signs of the eigenvalues, all non-invariant-manifold trajectories either diverge from or asymptotically approach the eigenvector planes, which therefore represent surfaces of attachment or separation. When the eigenvalues include a pair of complex conjugates, the single real eigenvector represents the axis of rotation for swirling flow. This list of linear vector field properties associated with invariant subspaces is far from complete, and is intended merely to illustrate the idea that a local linearized approximation of a vector field contains several intrinsic geometric loci which can be used as condition queries to determine the local topological features of a vector field.

Conceptually, one uses invariant manifolds as geometric representations of condition queries by mapping an element into the linearized field it induces, and determining if the image of the element intersects the invariant manifolds. In practice, since the invariant manifolds are lines and planes through the origin (i.e. vector subspaces) they can be represented by single tangent or normal vectors, and the intersection test reduces to one of parallelism or perpendicularity. That is, from any vector field one can derive a test vector field from the Jacobian eigenvectors or eigenvector plane normals, and the test vector field can be used in a relative directional query — perhaps selectively applied to its target vector field according to a PQR-space location-based "stencil". See Figures 4 and 5.

2.5 Other spaces

Vector component space, PQR space, and linearized vector spaces are just a few examples of countless derived spaces in which vector fields can be usefully plotted, and in which geometric representations of condition queries can be conveniently framed. The aforementioned spaces are particularly useful in these regards because they highlight generic topological and geometric features of three-dimensional vector fields, and because they contain symmetries or natural partitions that make possible simple and meaningful geometric representations of complex condition queries. Other spaces with these properties include curvature/torsion space, curvature/torsion/magnitude space, divergence/gradient magnitude/curl magnitude space, and eigenvalue space. Note that these spaces are not all three-dimensional.

For domain-specific vector field analyses, there may be natural domain-specific spaces in which vector fields can be informatively queried. Henze [2] discussed some "aerocentric" possibilities for two-dimensional flow fields.

Another class of possibilities relies upon transforming a target vector field into some other vector field. Condition queries "directly" on the derived vector field then serve "indirectly" as condition queries on the original target vector field, modulated by the transformation. For example, a derived vector field might be the gradient of the magnitude of a target vector field. A simple geometric condition query on the derived vector field asking for locations with high magnitude that are pointing North, is also a condition query on the target vector field asking for places where, locally, more Northerly vectors are larger than more Southerly ones.

2.6 Complex condition queries

Condition queries on vector fields can be arbitrarily complex. One can imagine posing condition queries built from geometric, topological, positional, relational, and temporal aspects of a target vector field location. A complex condition query can involve any kind of *logical composition* of basic vector field attributes. Since the emphasis here is on representing condition queries as geometric objects, it is helpful to imagine their logical composition in terms of Boolean models in solid model construction. As noted above, it is also sometimes useful to use the outcome of a condition query as a logical branch point (a `switch` statement) for further queries.

3 Examples

3.1 Velocity component space

For flow visualization, a primary target vector field for condition queries is the velocity field. It is informative to visualize the velocity field as a distribution of points in *velocity component space*. The mapping of velocity vectors into their component space is known as the *hodograph transform*.

Typically, a number of major flow features are revealed as large-scale distribution patterns in velocity component space; and there are generally a multitude of more subtle arrangements. For example, in the hemispherical cylinder dataset shown in Figure 1, one can identify the freestream, two primary vortices, and a prominent region of flow reversal. These features are also evident when the velocity vectors are normalized so that they all lie on the unit direction sphere.

Simple geometric partitionings of the velocity component space image can distinguish interesting and important regions of a flow. For instance, one could select the half of the direction sphere opposite the freestream to identify regions of flow reversal. In Figure 1, a selection of directions "steeper" than the freestream reveals areas where the flow is deflected up off the nose, and also where it is being swept upwards into the two primary vortices. One could refine the directional selection with a radial partition, for instance to show regions where the upward-diverted flow has been accelerated relative to the freestream. This example shows how data partitioning can be guided by the geometric structure of its image in a derived space, and also by the intrinsic geometric structure of the derived space itself.

3.2 Vortex cores

An important but somewhat ill-defined feature of many flows is the vortex, loosely described as a swirling region of flow. Much attention is given to vortex cores, loosely described as the axes of swirling flows. In some flows vortices are very compact structures, and the location of the core is a good indication of the location of the entire vortex; and even for flows with very "open" vortices, the cores can serve as a good indicator of overall topological structure.

3.2.1 Velocity and vorticity

One way to locate vortices is to look for regions where the velocity field is parallel to its own curl (see Figure 3). This is a nonuniform directional condition query with the velocity and vorticity fields playing the roles of target and test. In Beltrami flows velocity/vorticity parallelism holds everywhere, but in the more general case the condition is restricted to one-dimensional loci. We might call such space curves "Beltrami lines". If the velocity field is normalized, the Beltrami lines will be loci of maximum helicity; and this helicity condition is very nearly realized anyway in many physical flows.

3.2.2 Velocity gradient tensor

An important vortex core finder was popularized by Sujudi in [6]. As originally described, the algorithm locates regions where the velocity gradient tensor has complex conjugate eigenvalues, and also where the local velocity field is zero after it has been reduced by the velocity in the direction of the single real eigenvector. As noted by Roth and Peikert [5], the latter condition is equivalent to the velocity vector being parallel to the real eigenvector, and this in turn is equivalent to the condition of zero curvature in the local linearized velocity field.

We can interpret the "reduced velocity" algorithm in terms of two condition queries represented by statements of position in two three-dimensional derived spaces.

The requirement that the velocity gradient tensor has complex conjugate eigenvalues is met if and only if the tensor invariants fall in the appropriate regions of PQR-space (see Figure 2). The result of a query with this condition alone identifies regions with swirling flow, but also regions with shearing flow (see Figure 4). There exists

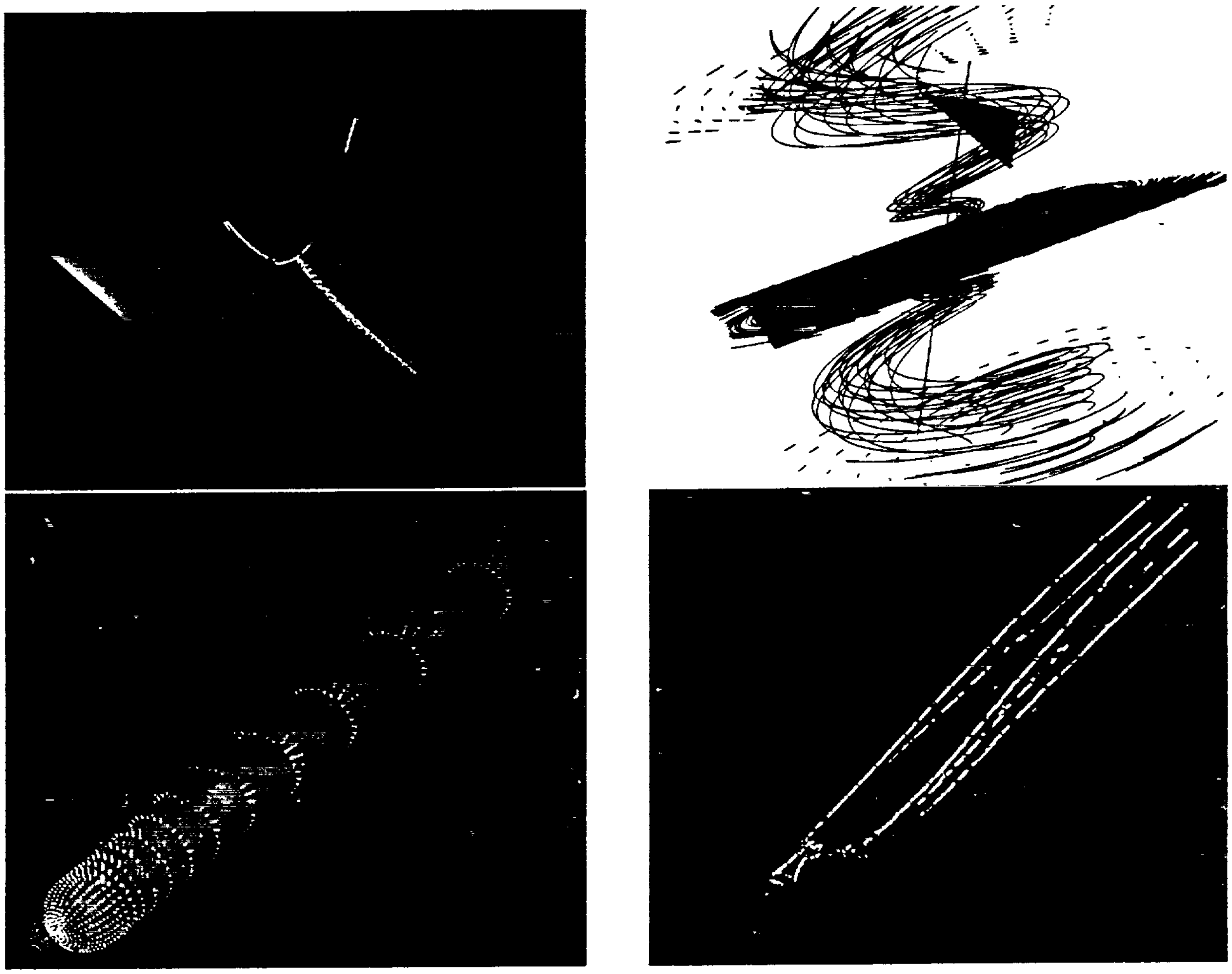

Figure 4: The two part condition query for vortex cores. (Top left) The first part of the query is represented by the regions in PQR-space which correspond to complex eigenvalues (notice the "jet" of points which extend into the complex region). (Bottom left) Results of the first query, revealing the high-shear boundary layer, and two "rails" of swirling flow extending along both sides of the top of the cylinder. (Top right) The second part of the query refines the first, querying the initial results for linear elements which are "spiked" by the single real eigenvector in the local linearized approximation to the flow. (Bottom right) Results of the second query, indicating line segments which are axes of the locally linearized swirling flow.

the possibility of refining the algorithm at this point, by restricting the PQR-space selection to one or the other of two complex regions, in order to select either inward-spiralling or outward-spiralling vortices.

After the PQR-space "topological prescreening", we can represent the zero reduced velocity condition geometrically as the single real eigenvector in the local linearized approximation of the vector field. Since the linear approximation is derived from the positions and velocity values of the vertices of a tetrahedron, one can locate the image of the tetrahedron in the linearized field. Vertex positions in the linearized flow field indicate precisely the vertex velocity values; interior points of the tetrahedral image indicate linearly interpolated values. If the image of a linear element intersects the real eigenvector — that is, if the image of a tetrahedron is "spiked" by the real eigenvector — then that element contains a segment of vortex core, whose precise location is easily determined by simple linear interpolation. The condition is illustrated in Figure 4. One can see graphically why this condition can be imposed by requiring the local velocity to be parallel to the real eigenvector. One can also see why the zero-curvature condition identifies local flow alignment with the real eigenvector, since in the linearized field, the real eigenvector provides the only uncurved trajectory in town.

3.2.3 Vorticity gradient tensor

One can represent the reduced velocity vortex core condition by means of a test vector field derived from the real eigenvectors of the velocity field Jacobian, and a PQR-space "stencil" guiding the applicability of a condition of parallelism. One can carry out precisely the same test with a test vector field derived from the real eigenvectors of the *vorticity* field — that is, one transforms the velocity field with the curl operator, then derives its real eigenvectors, and in places where the eigenvalues contain complex conjugates, one tests for parallelism *with the velocity field*. Sample results are shown in Figure 3. Note that while this algorithm would be hard to describe in reduced velocity terms, from the geometric viewpoint it is virtually identical to the reduced velocity strategy, and except for the additional requirement of a curl operator, the two algorithms require precisely the same geometric operations. (Actually, since $\boldsymbol{\Omega} = -\mathbf{1} \times \omega/2$, where $\boldsymbol{\Omega}$ is the antisymmetric part of ∇v and ω is the vorticity, the relationship to the reduced velocity algorithm can be demonstrated algebraically — and will be done so elsewhere.)

3.2.4 Higher-order vortex cores

Roth and Peikert [5] have described a "higher-order" vortex core finder, and have provided evidence that it locates curved vortex cores more accurately than do linear methods. Their algorithm reduces to a simple test of parallelism, between the velocity field and a derived test vector field $\mathbf{b}$. In steady flows, $\mathbf{b} = \mathbf{J}\mathbf{J}\mathbf{v} + \mathbf{T}\mathbf{v}\mathbf{v}$, where $\mathbf{v}$ is the target vector field, $\mathbf{J}$ is its Jacobian, and $\mathbf{T}$ is its Hessian (matrix of second derivatives.

3.3 Separation surfaces

"Separated flow" is evidently of great interest to aeronautical engineers, but despite the extensive literature on the subject it is hard to find any clear definition of the phenomenon. Kenwright [3] presented an operational definition of separation *lines* on two-dimensional surfaces, and Kenwright, Henze and Levit [4] discussed this algorithm and a closely related one explicitly in terms of vector field parallelism. Here we propose an operational definition for separation *surfaces* in three-dimensional flow fields, using geometric representations of condition queries.

The condition query seeking separation surfaces uses the velocity field as its target vector field, and has two parts. The first part is a "topological prescreen", represented by those loci in PQR-space where flow diverges from a single invariant manifold. These loci are simply the places which signify a single real positive eigenvalue of the Jacobian matrix. The two remaining eigenvalues can either be real and negative, or complex conjugates. The second part of the separation surface condition query is represented by the invariant manifold from which flow diverges in the linearized approximations of the vector field (at locations which have satisfied the topological test). If the image of a linear element in the linearized field intersects the divergent invariant manifold, then that element contains a section of separation surface, whose precise location is easily obtained by simple linear interpolation. The conditions and sample results for both the real and complex cases are shown in Figure 5.

4 Conclusions

By their nature, condition queries can reveal any regions with clearly definable characteristics in distributed data such as three-dimensional vector fields. These locations can range from arbitrarily defined "regions of momentary interest" to *bona fide* features, whose definitions themselves are the subject of theoretical concern. One benefit of considering feature detection strategies in terms of condition queries is that it forces one to immediately confront the operational definition of a proposed feature. One need only look at the aeronautics literature concerning vortices or separation surfaces, for example, to see the sorts of confusion and misguided efforts that arise when a clear operational definition of a "feature" is lacking.

Because condition queries can be so generally framed and applied, a systematic principle for their description and use can be immensely helpful from both a conceptual and a practical standpoint. It has been argued here that with respect to three-dimensional vector fields, representing condition queries geometrically in spaces derived from the field data provides just this sort of organizing theme. The geometric viewpoint permits an economy of description and conceptualization that is applicable across a wide variety of exploratory and analytic techniques. This same unification carries over to the actual computer implementations of these techniques, which need only support a handful of basic geometric operations to provide the foundation for a panoply of feature detection algorithms.

The examples here focused on flow fields, which have traditionally provided much of the impetus for scientific visualization of three-dimensional vector fields. Only a few very basic derived spaces were considered — namely, vector component space, PQR-space, and the locally linearized vector space — and only a few basic geometric representations were considered in those spaces — namely, points, lines, planes, spheres and cones, and the surfaces that make up the natural partitioning of PQR-space. With this small collection of shapes and spaces, we were able to operationally define critical points, isodirection lines, isonormal planes, isomagnitude loci, several existing vortex core detection algorithms and a new one, and a new separation surface algorithm. The actual implementation of all the queries described here relied primarily on a vector parallelism/perpendicularity finder, a vector magnitude tester, and a PQR-space locator.

That a geometric approach to condition queries on three-dimensional vector fields works so well should be no surprise since vector fields are, in the end, geometric objects. A great deal of the power of this approach comes from the fact that complex, domain-specific details are pushed into the various derived space mappings, allowing very involved conditions to be described in simple geometric terms, and reducing what might be tedious or difficult evaluation of conditions to a few simple and routine geometric operations.

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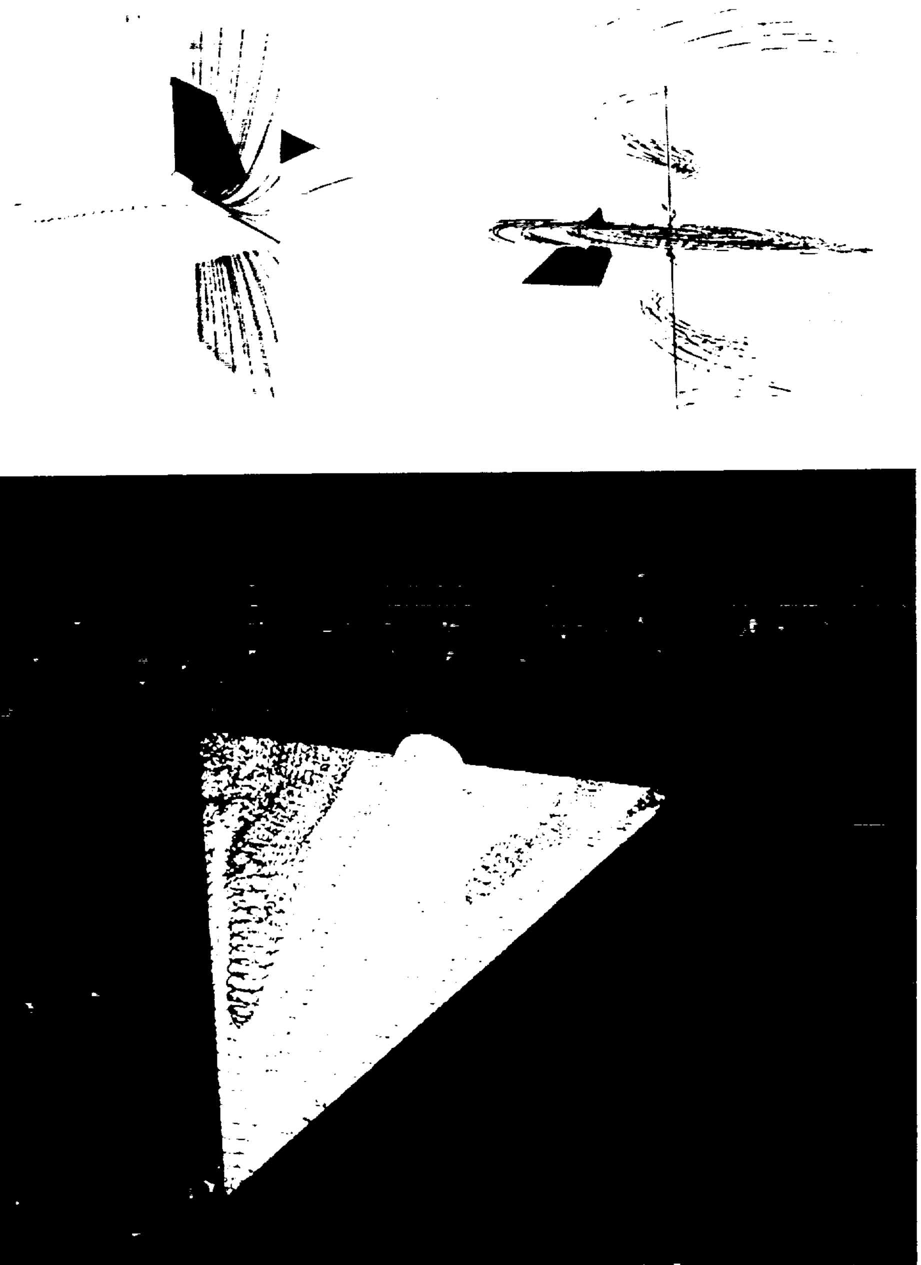

Figure 5: Geometric representations and results of a condition query for separation surfaces in a three-dimensional flow field around a rolling delta wing at 30 angle of attack. The red tetrahedron (top left) and points (bottom) indicate the intersections of simulation domain linear elements with the node invariant manifold of a local linear approximation to the flow with one positive and two negative real eigenvalues. The green tetrahedron (top right) and points (bottom) indicate the intersections of linear elements with the single invariant manifold of a local linear approximation to the flow with complex conjugate eigenvalues. Intersections of the green tetrahedron with the single real eigenvector in the complex case represent vortex cores (not shown here) which are associated with the regions above the wing enclosed by green points.